Rates of Change

and

Higher-Order Derivatives

Higher Order Derivatives

first derivative	**y'**	**$\frac{dy}{dx}$**	**$\frac{d}{dx}[f(x)]$**
second derivative	y''	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}[f(x)]$
	y'''	$\frac{d^3y}{dx^3}$	
fourth	$y^{(4)}$		

$y = 3x^5 + 4x^4 + 4x^2$ Find $\frac{d^2y}{dx^2}$

$y' = 15x^4 + 16x^3 + 8x$

$\boxed{y'' = 60x^3 + 48x^2 + 8}$

$y = \sin x$ y'''

$y' = \cos x$

$y'' = -\sin x$

$\boxed{y''' = -\cos x}$

$y = 3x^4 - 3x$ Find $\dfrac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = 36x^2$$

$$y = -x^4 + x^2 \quad \text{Find } \frac{d^3y}{dx^3}$$

$$\frac{d^3y}{dx^3} = -24x$$

$f(x) = 4x^5 + 5x^4 + 3x^2$ Find f'''

$240x^2 + 120x$

Rates of Change

speed (rate): $\frac{\textbf{distance}}{\textbf{time}}$

Scalar quantity
(no direction)

velocity (speed in a given direction):

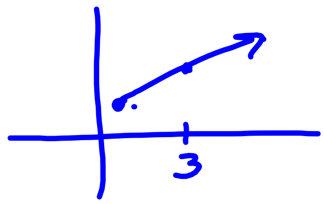
Vector quantity

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta s}{\Delta t}$$

Acceleration:

vector quantity

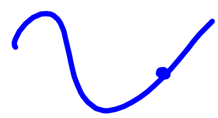
$$\frac{\textbf{change in velocity}}{\textbf{time}}$$



vel / accel.

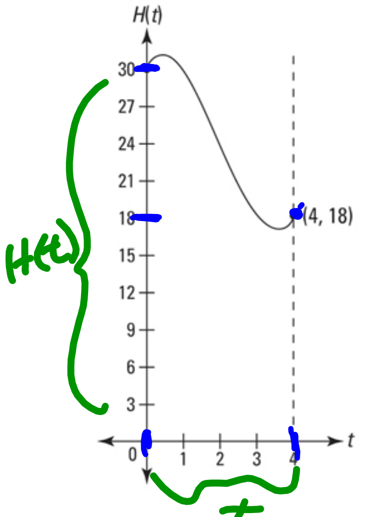
up / right = Positive

down / left = negative



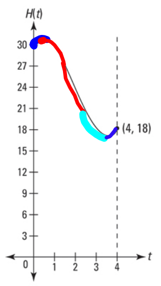
A yo-yo moves straight up and down. Its height above the ground, as a function of time, is given by the function $H(t) = t^3 - 6t^2 + 5t + 30$

t is in seconds and H(t) is in inches. At t=0, it's 30 inches above the ground, and after 4 seconds, it's at a height of 18 inches.



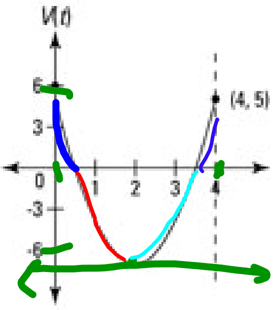
The yo-yo's height, ... 0 to 4 seconds.

$$H(t) = t^3 - 6t^2 + 5t + 30$$

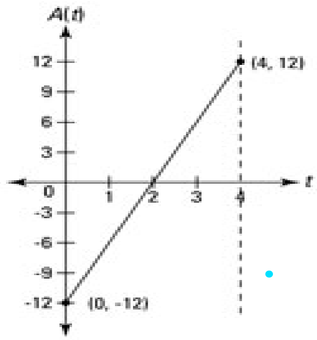


The yo-yo's height, from 0 to 4 seconds.

$H(t) = t^3 - 6t^2 + 5t + 30$



$v(t) = 3t^2 - 12t + 5$



$A(t) = 6t - 12$

When position is written w/ respect to time:

Velocity is the 1^{st} derivative of position

Acceleration is the 2^{nd} derivative of position

If a billiard ball is dropped from a height of 100 feet, its height s, in feet, at time t, in seconds, is given by the position function

$$s = -16t^2 + 100$$

Find the average velocity over each of the following time intervals.

a. [1,2] b. [1, 1.5] c. [1, 1.1]

$-16(1)^2+100 = 84$

$-16(2)^2+100 = 36$

$\frac{36-84}{2-1} = -48$ ft/sec

-40 ft/sec

-33.6 ft/sec

s(t) is the height or position function

$$v(t) = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t} = s'(t)$$

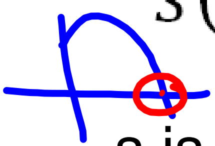
When a function gives position with respect to time:

Velocity is the 1st derivative of position

Acceleration is the 2nd derivative of position

At time t=0, a diver jumps from a platform board that is 32 feet above the water. The position of the diver is given by

$$s(t) = -16t^2 + 16t + 32$$



s is measured in feet, t in seconds

a. When does the diver hit the water?

$-16(t^2 - t - 2)$

$-16(t+1)(t-2)$

$-1, 2$

2 seconds

b. What is the diver's velocity at impact?

$s'(t) = v(t) = -32t + 16$

$= -32(2) + 16$

$= -48$ ft/sec

Higher-Order Derivatives

You can obtain an acceleration function by differentiating a velocity function.

$s(t) =$ position function

$v(t) = s'(t)$ velocity function

$a(t) = v'(t) = s''(t)$ acceleration function

first derivative	y'	$\frac{dy}{dx}$	$\frac{d}{dx}[f(x)]$
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fourth	$y^{(4)}$		